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On the 29th May 2018, Alex became the first PhD student under the ISP subnetwork Eastern Africa Universities Mathematics Programme to defend a Swedish PhD at Mälardalen University, Västerås, Sweden. He always had a deep interest in the interplay between dynamical systems and operator algebras and in this thesis, He treated commutativity which is a fundamental topic in mathematics, physics, engineering and many other fields. Two processes are said to be commutative if the result of the processes does not depend on the order in which the processes are applied. Commutativity of addition can be observed when paying for items at the counter by cash. No matter the order in which bills are handed over, they always give the same total, whereas washing and then drying clothes, and drying and then washing produce markedly different results.

An example of the importance of commutativity comes from signal processing. Signals pass through filters (often called operators on a Hilbert space by mathematicians) and commutativity of two operators corresponds to having the same result even when filters are interchanged. Many important relations in mathematics, physics and engineering are represented by operators satisfying a number of commutation relations. This means that the operators do not actually commute but there is an explicit relation for the difference of the two possible products of the operators.

Part of his PhD thesis treats commutativity of monomials of pairs of operators satisfying certain commutation relations. He considered products of powers of the operators, called monomials, and derive commutativity conditions of the said monomials. He shows that this is related to the existence of periodic points of certain one-dimensional dynamical systems. In the other part of the thesis, he treats maximal commutative subalgebras of crossed products of algebras piecewise constant functions with the integers. By the crossed product of an algebra with the integers he meant a generalization of Laurent polynomials with coefficients from the algebra. He describes the commutant (set of elements that commute with a given set) and the center (set of elements that commute with the whole algebra) in a number of cases. Finally, turns his attention to Ore extensions. By an Ore extension of a ring, meaning a generalization of polynomials with coefficients from the ring, he describes the commutant of the coefficient algebra for the Ore extension of the algebra of functions with finite support on a set.