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**Thesis title:** The use of Splines for solving ill-posed problems, with application to the Cauchy problems for the Heat and Helmholtz equations

**Abstract**

**Inverse heat conduction:** The Cauchy problem for the heat equation is a model of situation where one seeks to compute the temperature, or heat-flux, at the surface of a body by using interior measurements. The problem is well-known to be ill-posed, in the sense that measurement errors can be magnified and destroy the solution, and thus regularization is needed. In previous work it has been found that a method based on approximating the time derivative by a Fourier series works well (Berntsson, 1999; Eldén, Berntsson, & Reginska, 2000). However, in this study it is not reasonable to assume that the temperature is periodic which means that additional techniques are needed to reduce the errors introduced by implicitly making the assumption that the solution is periodic in time. Thus, as an alternative approach, I instead approximate the time derivative by using a cubic smoothing spline. This means avoiding a periodicity assumption which leads to slightly smaller errors at the end points of the measurement interval. The spline method is shown to satisfy similar stability estimates as the Fourier series method. Numerical simulations have proved that both methods work well, and provide comparable accuracy, and also that the spline method gives slightly better results, compared to the Fourier method.

**Helmholtz Equation:** The Cauchy problem for the Helmholtz equation, has also been considered where it is dened in a rectangular domain. The Cauchy data are prescribed on a part of the boundary and the aim was to find the solution in the entire domain. The problem occurs in applications related to acoustics and is illposed in the sense of Hadamard (Hadamard 1923). In this study, I have considered regularizing the problem by introducing a bounded approximation of the second derivative by using Cubic smoothing splines. A bound for the approximate derivative has been derived and I have shown how to obtain stability estimates for the method. Numerical tests show that the method works well and can produce accurate results. It has been demonstrated that the method can be extended to more complicated domains. The regularization method based on splines for the solution to ill-posed problems in general, that was developed in this study with reference to (Nanfuka, Berntsson, & Kakuba, 2021), has also been applied to an industrial setting for solving the Inverse heat conduction problem. Both experimental data and simulated data have been used for this purpose.

**Application:** Here the splines method, based on approximating the time derivative in the heat equation by using cubic smoothing splines, has been used. The problem is ill-posed and the method involves a parameter λ that controls the smoothness of the computed derivatives, and acts as a regularization parameter. When applied to actual measured data from an application where a steel slab is heated in a laboratory scale furnace, the method gives slightly optimal results for the approximation of the surface temperature of steel, during the steel treatment process. I recommend that the developed regularization method, based on a cubic smoothing spline, can effectively and efficiently be used for the solution to 2-dimensional inverse problems ( Boundary value problems), that occur in the engineering, medical and applied science fields. In future the method can be modified to handle 3-dimensional problems.